Exercise 81

Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

Solution

Take the derivative of f.

$$f'(x) = \begin{cases} 2x & \text{if } x \leq 2 \\ m & \text{if } x > 2 \end{cases}$$

For f to be differentiable everywhere, it and its derivative must be continuous at the endpoints of every interval.

$$\text{At } x = 2: \quad x^2 = mx + b \qquad \rightarrow \qquad (2)^2 = m(2) + b \tag{1}$$

$$\text{At } x = 2: \quad 2x = m \qquad \rightarrow \qquad 2(2) = m \tag{2}$$

Solve equations (1) and (2) for m and b.

$$m = 4 \qquad b = -4$$